## Impedance Effects in the PS2

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#### Introduction

Estimates of the effect in the PS2 of the resistive wall impedance on transverse coupled bunch instability, the transverse mode coupling instability, and the longitudinal microwave instability were given at the Napa LARP workshop.

The coupled bunch instability growth time estimate was 60 turns (SS, at injection); the single bunch instability thresholds were comfortably high—though the major contributors to impedance are still missing from the calculations.

Here we will discuss:

- The PS2 beam pipe will be made of stainless steel. Should it be coated with a thin layer ( $\sim$  20  $\mu$ m or  $\sim$  2  $\mu$ m) of copper?
- We include the longitudinal space charge impedance in the microwave instability estimate.
- Finally, we discuss where we go from here.

#### Selected PS2 Parameters

Parameter	Value	Units
Circumference, C	1346.4	m
Chamber half apertures, $a_x$ by $a_y$	6 by 3.5	cm
Initial, final energies, $E_0$ , $E_f$	4, 50	GeV
Bunch population, $N_b$	4.2	10 <sup>11</sup>
Average current, I	2.7	Amp
Long. emittance $(4\pi\sigma_t\sigma_\delta)$ , $\epsilon_I$	0.6	eV-s
Norm. emittances $\gamma \epsilon_x = \gamma \epsilon_y$	3	π μm
Rms bunch length, $\sigma_{t0}$ , $\sigma_{tf}$	3.8, 1	ns
Rms rel. energy spread, $\sigma_{\delta 0}$ , $\sigma_{\delta f}$	3.2, 1	$10^{-3}$
Transition gamma, $\gamma_t$	35i	
Slippage factor, $\eta_0$ , $\eta_f$	-0.037, -0.0012	
Synchrotron tune, $v_{s0}$ , $v_{sf}$ ,	18, 0.8	$10^{-3}$
Vertical tune, $v_y$	8.2	
Average beta function, $\bar{\beta}_y$	31	m

# Transverse Coupled Bunch Instability

- The main contribution to the growth rate of the coupled-bunch instability is normally the resistive wall impedance
- The growth rate of the fastest growing mode in a round, thick-walled pipe (see e.g. A. Wolski, et al, LBNL-59449, Feb. 2006):

$$\Gamma = \frac{c}{4\gamma} \frac{m_e I}{m_p I_A} \sqrt{\frac{\mathcal{C}}{1 - [\nu_y]}} \langle \beta_y A_y \rangle$$

with

$$A_y = \frac{4}{\pi^{1/2}b^3}\sqrt{\frac{1}{Z_0\sigma_c}}$$

• For stainless steel at injection  $\Gamma=3650~s^{-1}$ , at extraction  $\Gamma=360~s^{-1}$ ; the growth times are equivalent to respectively 60., 625. turns

## Transverse Mode Coupling Instability

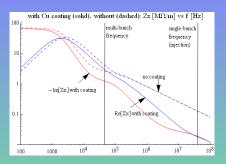
 The threshold can be approximated by (see S. Krinsky, BNL 75019-2005-IR)

$$\frac{N_{th}}{N_b} \sim 0.7 \frac{4\pi \mathcal{E} \nu_s}{e^2 N_b \bar{\beta}_y k_y \mathcal{C}} \ ,$$

with the vertical kick factor  $k_y = \langle V_y \rangle / e N_b C$ .

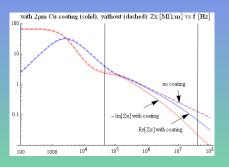
- for stainless steel at injection,  $N_{th}/N_b = 16$ ., at extraction 8.7.
- resistive wall is just one of many components in impedance budget

#### Transverse Impedance



Vertical impedance of the PS2 beam pipe, assuming 2 mm thick stainless steel, when it is (solid)/ is not (dashed) coated with 20  $\mu$ m of Cu. The calculation follows the cylindrically-symmetric formalism of Burov and Lebedev (EPAC 2002). The vertical lines locate the frequencies important for the coupled bunch instability and the mode coupling instability.

## Impedance-2 μm Coating



Vertical impedance of the PS2 beam pipe, assuming 2 mm thick stainless steel, when it is (solid)/ is not (dashed) coated with 2  $\mu$ m of Cu.

# Impedance (Cont'd)

- Coupled bunch instability: the important frequency is  $f=[\nu_y]f_{rev}=45$  kHz. Here  $Z_y=(4.1-2.6i)$  M $\Omega/\text{m}$  (no coating),  $Z_y=(3.0-1.2i)$  M $\Omega/\text{m}$  (with 20  $\mu$ m coating).
- Mode coupling instability (a single bunch instability): the important frequency is  $k \sim 1/\sigma_z$ , which at injection means f=40 MHz, at extraction f=155 MHz. Note that skin depth  $\delta(40 \ MHz)=10 \ \mu m$  (and less at extraction).  $20 \ \mu m$  coating reduces impedance here by factor  $\sim \sqrt{\sigma_{Cu}/\sigma_{SS}} \sim 7$ . But resistive wall impedance likely a small part of total impedance budget.
- Microwave instability: similar to mode coupling instability, but driven by longitudinal impedance. Again coating shields the impedance effectively, but again resistive wall impedance likely a small part of total impedance budget.

## Microwave Instability

Boussard criterion:

$$\frac{N_{th}}{N_b} \lesssim \frac{(2\pi)^{3/2} \sigma_z |\alpha| \mathcal{E} \sigma_\delta^2}{e^2 c N_b |Z/n|} ;$$

this criterion gives a very rough estimate of the threshold to the microwave instability.

• Resistive wall impedance:

$$\frac{Z}{n} = (1-i)\frac{\mathcal{C}}{2\pi b} \frac{1}{\delta_s \sigma_c} \frac{\omega_0}{\omega} ;$$

in Boussard criterion evaluate at  $\omega = c/\sigma_z$ .

• Space charge impedance:

$$\frac{Z}{n} \sim i \frac{4\pi}{\gamma^2 c} \ln \left( \frac{a_y}{\sigma_y} \right) .$$

- for PS2 with SS, resistive wall impedance  $|Z/n| = 0.51 \Omega$  (injection), = 0.26  $\Omega$  (extraction).
- space charge component of impedance: |Z/n| = 42.  $\Omega$  (injection), = 0.43  $\Omega$  (extraction).
- summing the two contributions gives:  $N_{th}/N_b = 31$ . (at injection), = 40. (at extraction).

# Where we go from here

We are missing the geometric impedance contributions, from objects such as :

- RF cavities
- BPMs-stripline type
- bellows
- injection and extraction kickers
- collimators
- flanges, pumping slots...

Once we have the number and shapes of the objects in the PS2 we can estimate the total impedance of the ring.

Once we have such an impedance model, we can perform stability studies through tracking and through solution of the Vlasov Equation.

#### Discussion

- For the PS2 a 20 μm layer of Cu will effectively shield the resistive wall component of the impedance for the transverse mode coupling instability and the microwave instability. However, the rw component is likely a small part of the total impedance budget in both cases.
- For the coupled bunch instability the rw component is dominant, with the instability growth time on the order of 60 turns (at injection). Shielding reduces  $|Z_y|$  by 33%. The effect on the instability needs more analysis (though is likely small).
- A 2 μm layer of Cu will have almost no effect on multi-bunch instability.
- Resistive wall calculations were performed assuming round geometry. Flat geometry calculations can and will be done.
- The next step in the impedance calculations for the PS2 will be to estimate the contribution of the geometric impedance sources.